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Review of power curve modelling for wind turbines

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ABSTRACT

Currently, variable speed wind turbine generators (VSWTs) are the type of wind turbines most widely installed. For wind energy studies, they are usually modelled by means the approximation of the manufacturer power curve using a generic equation. In literature, several expressions to do this approximation can be found; nevertheless, there is not much information about which is the most appropriate to represent the energy produced by a VSWT. For this reason, in this paper, it is carried out a review of the equations commonly used to represent the power curves of VSWTs: polynomial power curve, exponential power curve, cubic power curve and approximate cubic power curve. They have been compared to manufacturer power curves by using the coefficients of determination, as fitness indicators, and by using the estimation of energy production. Data gathered from nearly 200 commercial VSWTs, ranging from 225 to 7500 kW, has been used for this analysis. Results of the analysis presented in the paper show that exponential and cubic approximations give the higher R^2 values and the lower error in energy estimation. With the approximate cubic power curve quite high values of R^2 and low errors in energy estimation are achieved, which makes this kind of approximation very interesting due to its simplicity. Finally, the polynomial power curve shows the worst results mainly due to its sensitivity to the data given by the manufacturer.

Contents

1. Introduction

The power curve of a WTG is obtained by the manufacturers from field measurements of wind speed and power, apart from environmental values (temperature, pressure and relative

Abbreviations: WTG, wind turbine generator; VSWT, variable speed wind turbine

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Nomenclature

v	wind speed in m/s
A	rotor area in m^2
ρ	air density in kg/m^3
$p(v)$	electric power generated by the wind turbine in W
C_p	power coefficient
$p_w(v)$	power in W associated to a wind speed
v_{ci}	cut-in wind speed in m/s
v_{co}	cut-out wind speed in m/s
v_r	rated wind speed in m/s
P_r	rated power in W
$q(v)$	non-linear part of a power curve
$q'(v)$	non-linear part of fitted power curve
E	energy density in W/m^2
E'	energy density obtained from fitted curve
$f(v)$	probability in p.u. associated to a wind speed v
N	number of values in manufacturer power curve
v_I	discretised value of cut-in speed
v_O	discretised value of cut-out wind speed
v_R	discretised value of rated wind speed
f_{rj}	relative frequency associated to each wind speed v_j
C_1, C_2, C_3	coefficients of polynominal approximations
K_p, β	coefficients of exponential approximation
$C_{p,eq}$	coefficient of cubic approximation
$C_{p,max}$	maximum value of effective power coefficient
J	index for least square optimisation
$\xi\{\cdot\}$	mean function
R^2	coefficient of determination
ε	error of energy density in %

humidity). The measurements are usually averaged and normalised to a reference air density using normalised procedures [1]. The resulting discrete values of the power curve for a determined WTG are usually available from manufacturers, and they can be used for studies involving energy evaluation.

Nevertheless, for the sake of generality, it is common that a generic equation for modelling the power curve will be preferred in studies about WTG modelling [2–5], analysis of wind energy potential [6], site matching [5,7–9], cost modelling [10,11], etc. In this context, the use of an equation for representing a power curve and the obtention of its parameters becomes an important issue. The main problem derived from using a generic equation is the fact that is hard to know how this equation will accurately represent any commercial WTG.

In the first term, the power curve of a WTG can be estimated using the power curve coefficient (C_p) from the turbine blade parameters (blade design, tip speed ratio and pitch angle) [4], the rotor dimensions and the reference air density. For example in [12] the power coefficient is calculated through an expression that links the blade radius, blade design constant and wind turbine shaft angular speed with the power coefficient. In [11] an expression is proposed for the approximation of C_p, considering a rated power coefficient, rated wind speed and a parameter expressing the operation range of wind speed. The shortcomings of using the models proposed in [11] and [12] are that they depend on some technical factors of the wind turbines which are difficult to obtain from the manufacturers.

Another way to approximate the power curve is presented in [2], where power curves are approximated by means of fitting techniques, like least squares or cubic spline interpolation. Although pretty accurate fits are achieved, the resulting power curve equations are quite complex, which makes it difficult to find a generic expression.

To overcome the problems depicted above, the power curve of WTGs is usually represented by means of a polynomial power curve [13–15] or by means of an exponential power curve or its simplifications [16]. Their parameters can be derived from manufacturer data or by fitting the manufacturer power curve. However, although these expressions are widely used, there is little evidence of how these curves fit with real WTGs [13–18]. For this reason, in this paper is presented a study of the power curve models taking into account a database with manufacturer information from nearly 200 variable speed wind turbines (VSWT). Only VSWTs have been considered in this paper because they represent the state of art of commercial WTGs installed at present. The most important wind turbine manufacturers have been included in this database.

In order to analyse which are the most appropriate equations to approximate power curves, it is also presented a critical comparison of the fitted power curves considering the coefficient of determination R^2, as a measure of goodness of fit, and the difference between the estimations of energy density when the fitted and the manufacturer power curves are used.

The paper is organised as follows. Section 2 presents the identification of the main features of the power curve. Section 3 summarises the most typical models used for the representation of the power curves. Section 4 shows the database used for the characterisation of the power curves including the main characteristics of the wind turbines. In Section 5, the results of the fitting methods and indicators of fitness are presented. Finally, conclusions are given in Section 7.

2. Energy evaluation and power curve

The available power of the wind that crosses the rotor of a wind turbine can be obtained from

$$p_w(v) = \tfrac{1}{2}A\rho v^3 \tag{1}$$

where $p_w(v)$ is the power in W associated to a wind speed v in m/s, A is the rotor area in m^2 and ρ is the air density (typ. 1225 kg/m^3 [1]). This power is related to power generated by a wind turbine by means of the power coefficient

$$C_p(v) = p(v)/p_w(v) \tag{2}$$

where $p(v)$ is the power generated by the wind turbine in W, C_p is the power coefficient that is related to the blade design, the tip angle and the relationship between rotor speed and wind speed. The maximum theoretical value of power coefficient, known as the Betz limit, is 0.593 (16/27). However, this value is not achievable with real turbines and its maximum value is normally around 0.5. The power coefficient can be obtained from the manufacturer data, as a consequence, mechanical and electrical losses are usually included in the coefficient value as well as the aerodynamic behaviour of blades.

The power delivered by a wind turbine is usually represented through its power curve, where a relation between the wind speed and the power is established. For the VSWTs, this relationship can be expressed in the following way:

$$p(v) = \begin{cases} 0 & v < v_{ci} \text{ or } v > v_{co} \\ q(v) & v_{ci} \le v < v_r \\ P_r & v_r \le v \le v_{co} \end{cases} \tag{3}$$

where $p(v)$ is the electric power in W, v_{ci} is the cut-in wind speed in m/s, v_{co} is the cut-out wind speed in m/s, v_r is the rated wind speed in m/s, P_r is the rated power in W and $q(v)$ is the non-linear relationship between power and wind speed (see Fig. 1).

The shape of the non-linear part is related to the control strategy of extracting as much power as possible from the wind. This is why it is roughly represented by a cubic expression [17].

The zones of the power curve defined by cut-in, rated and cut-out wind speeds are clearly specified in (3). Nevertheless, it must be kept in mind that the power curve is obtained from mean values of a set of measurements [1]. This is the main explanation for the typical smooth shape of the power curve. Consequently, the limits shown in (3) are not as clearly defined in manufacturer power curve as those shown in the mentioned equation.

The energy density E in W/m^2 for a specific wind site and a wind turbine can be obtained by using the power curve and the probability distribution function of wind speed

$$E=\frac{1}{A}\int_{v_{ci}}^{v_{co}} p(v)f(v)\,dv \tag{4}$$

where $f(v)$ represents the probability in p.u. associated to the wind speed v [19]. The discrete version of this equation can be written as

$$E=\frac{1}{A}\sum_{j=1}^{N} fr_j p(v_j)=\frac{1}{A}\sum_{j=I}^{R-1} fr_j p(v_j)+\frac{1}{A}P_r\sum_{j=R}^{O} fr_j \tag{5}$$

where N is the number of power curve values, $v_I=v_{ci}$ is the cut-in speed, $v_O=v_{co}$ is the cut-out wind speed, $v_R=v_r$ is the rated wind speed and f_{rj} is the relative frequency associated to each wind speed v_j that can be obtained from the histogram of wind speeds.

3. Power curve characterisation

The most typical mathematical equations for representing the non-linear part $q(v)$ of a power curve are:

- Polynomial power curve.
- Exponential power curve.
- Cubic power curve.
- Approximate cubic power curve.

3.1. Polynomial power curve

In the polynomial power curve approximation, a second degree polynomial is used to fit $q(v)$ [13–15]

$$q(v)=C_1+C_2 v+C_3 v^2 \tag{6}$$

where C_1, C_2 and C_3 are coefficients calculated from v_{ci}, P_r and v_r.

$$C_1=\frac{1}{(v_{ci}-v_r)^2}\left[v_{ci}(v_{ci}+v_r)-4v_{ci}v_r\left(\frac{v_{ci}+v_r}{2v_r}\right)^3\right]$$

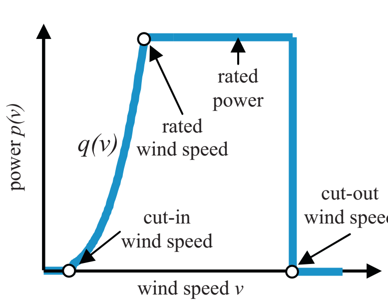


Fig. 1. Representation of the power curve.

$$C_2=\frac{1}{(v_{ci}-v_r)^2}\left[4(v_{ci}+v_r)\left(\frac{v_{ci}+v_r}{2v_r}\right)^3-3v_{ci}-v_r\right]$$

$$C_3=\frac{1}{(v_{ci}-v_r)^2}\left[2-4\left(\frac{v_{ci}+v_r}{2v_r}\right)^3\right]$$

3.2. Exponential power curve

When an exponential power curve is used to model a VSWT power curve, the non-linear curve $q(v)$ is approximated by using [16]

$$q(v)=\tfrac{1}{2}\rho A K_p\left(v^{\beta}-v_{ci}^{\beta}\right) \tag{7}$$

where K_p and β are constants.

3.3. Cubic power curve

A typical simplification of the expression shown in (7) can be obtained supposing v_{ci} equal to zero and β equal to three. As a result, a cubic power curve approximation, that is similar to (1), is obtained [5,20]

$$q(v)=\tfrac{1}{2}\rho A C_{p,eq} v^3 \tag{8}$$

where $C_{p,eq}$ is a constant equivalent to the power coefficient.

3.4. Approximate cubic power curve

An approximation of (8), called approximate cubic power curve, can be obtained by assuming $C_{p,eq}$ equal to the maximum value of effective power coefficient ($C_{p,max}$). The term "effective" means that mechanical and electrical losses are included in this coefficient. The resulting equation is

$$q(v)=\tfrac{1}{2}\rho A C_{p,\max} v^3 \tag{9}$$

4. Wind turbine characteristics

The first step in order to compare the power curves is to gather VSWT data from different WTG databases and information from manufacturers [21–26]. A database of 187 VSWTs, including parameters like power curve data and type of generator, has been used (see Appendix B). As an example, the representation of the power curves can be seen in Fig. 2. The values of cut-in, cut-out, rated wind

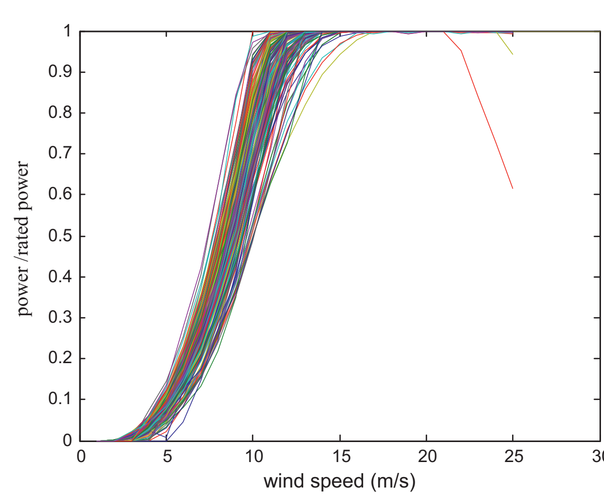


Fig. 2. Representation of all power curves in database.

speed, maximum hub height and the type of generator are presented in Figs. 3–5. Finally, the VSWT technologies are shown in Fig. 6.

As a first analysis, it can be concluded that the typical values for cut-in wind speeds are lower than 5 m/s ($v_{ci} < 5$ m/s), for cut-out wind speeds are higher than 15 m/s ($v_{co} > 15$ m/s), and rated wind speeds lie between 8 m/s and 18 m/s (8 m/s $< v_{cr} <$ 18 m/s) as can be seen in Fig. 5.

Aiming to guarantee the consistency of data, v_{ci}, v_r and v_{co} have been directly obtained from the manufacturer power curve.

5. Power curve modelling

5.1. Power curve fitting

The main objective of this paper is to determine which of the equations presented in Section 3 are the most appropriate to represent the behaviour of the power curves given by the manufacturers. For this reason the proposed equations to be evaluated are the polynomial (6), the exponential (7), the cubic (8) and the approximate cubic (9) power curves. The parameters for the calculation of polynomial power curve, C_1, C_2 and C_3 in (6), and the parameter of approximate cubic power curve, $C_{p,max}$ in (9), can be obtained directly from the manufacturer data.

In the other hand, the parameters of the exponential power curve, K_p and β in (7), and the parameter in the cubic power curve, $C_{p,eq}$ in (8), must be calculated using a curve fitting method. In this case, it has been used a least squares one which minimises the following index:

$$J = \sum_{j=1}^{R-1} (q(v_j) - q'(v_j))^2 \tag{10}$$

where $q(v)$ represents the non-linear part of manufacturer power curve (see Fig. 1) and $q'(v)$ is its corresponding fitted curve.

The index J has been minimised using a Nelder–Mead simplex method implemented in MATLAB [27].

5.2. Goodness of fit

Two indicators of goodness of fit have been selected for the comparison of the power curves: the coefficient of determination R^2 and the mean energy production calculated by using different mean wind speeds.

5.2.1. Coefficient of determination R^2

The coefficient of determination R^2 is used to compare the results of the manufacturer power curve with the power curves

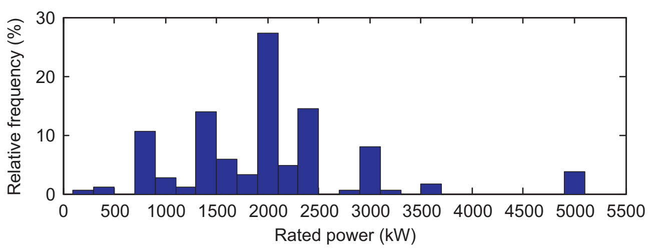


Fig. 3. Histogram of rated power.

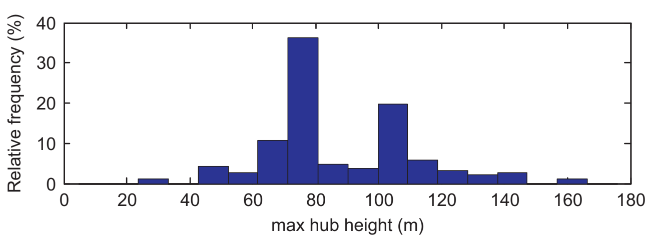


Fig. 4. Histogram of maximum hub height.

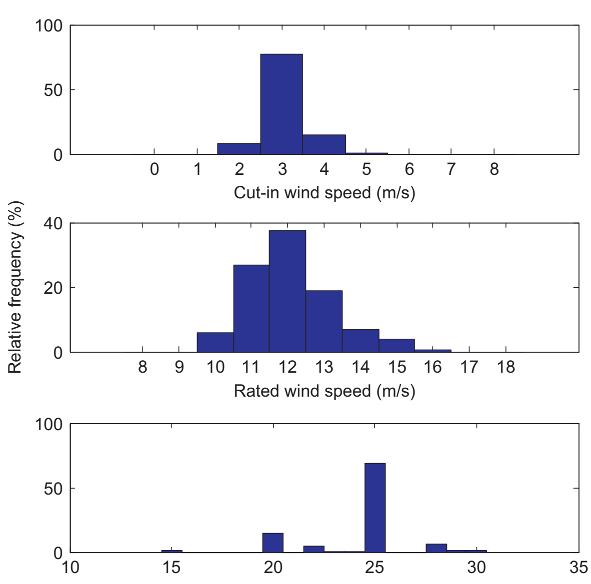


Fig. 5. Histogram of cut-in, rated and cut-out wind speeds.

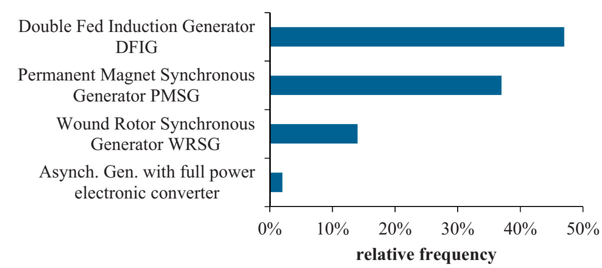


Fig. 6. Histogram of VSWT technologies.

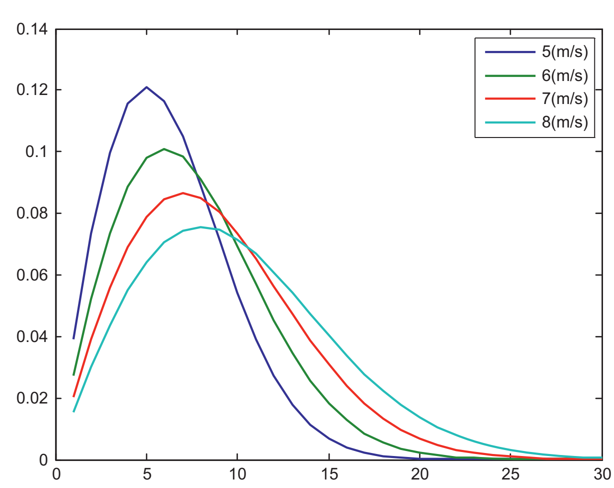


Fig. 7. Rayleigh probability density function of the mean wind speeds (5, 6, 7, and 8 m/s).

obtained with (6)–(9). This coefficient can be defined as

$$R^2 = 1 - \frac{\sum_{j=I}^{R-1} (q(v_j) - q'(v_j))^2}{\sum_{j=I}^{R-1} (q(v_j) - \xi\{q(v_j)\})^2} \qquad (11)$$

where $\xi\{q(v)\}$ is the mean of the non-linear part of the manufacturer power curve.

In this case, the R^2 coefficient is closely related to the expression of index J in (10) used during the curve fitting process. Thus, for the exponential and cubic power curves the R^2 values are supposed to be the highest.

5.2.2. Energy production

Another selection criteria considered, in order to determine which is the most appropriate equation that fits the manufacturer power curves, is the relative error between the energy calculated from manufacturer power curve and the energy obtained from the fitted power curves. For this calculation, a set of Rayleigh PDF with different mean wind speeds (5, 6, 7 and 8 m/s) has been used. Its representation can be seen in Fig. 7 [19]. These mean wind speeds have been chosen because they are typical in wind energy installations, and with them can be achieved the common utilisation times between 2200 and 3500 h/year [17].

The error of energy density ε in % is calculated by means of the following expression:

$$\varepsilon = \frac{E' - E}{E} \times 100 \qquad (12)$$

where E is the energy density obtained from manufacturer power curve and E' is the energy density obtained from the fitted power

Table 1
Manufacturer power curve.

Wind speed (m/s)	Power (kW)	Wind speed (m/s)	Power (kW)	Wind speed (m/s)	Power (kW)
1	0	10	1580	19	2050
2	3	11	1810	20	2050
3	25	12	1980	21	2050
4	82	13	2050	22	2050
5	174	14	2050	23	2050
6	321	15	2050	24	2050
7	532	16	2050	25	2050
8	815	17	2050		
9	1180	18	2050		

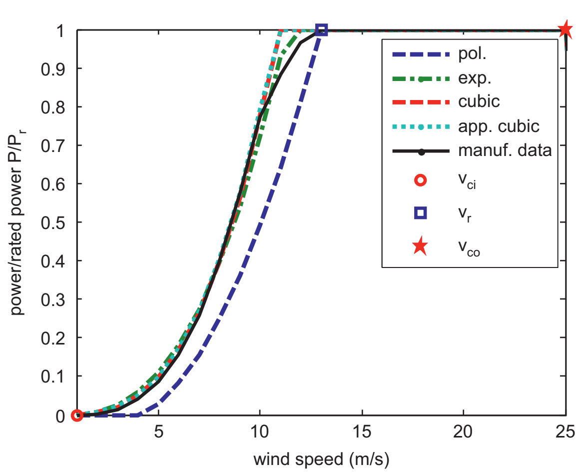


Fig. 8. Power curve and its approximations.

Table 2
Parameters of approximation equations for a 2 MW VSWT.

Approximation	Parameters		
Polynomial	C_1	C2	C3
	0.0408	−0.0504	0.0095
Exponential	K_p	β	
	0.899	2.706	
Cubic	$C_{p,eq}$		
	0.490		
Approx. Cubic	$C_{p,max}$		
	0.500		

Table 3
Goodness of fit for a 2 MW VSWT.

Approximation	Coeff. R^2	Error of energy density ε (%)			
		5 m/s	6 m/s	7 m/s	8 m/s
Polynomial	0.866	−31.5	−24.8	−19.7	−16.1
Exponential	0.995	2.9	1.7	1.1	0.8
Cubic	0.992	3.6	3.0	2.5	2.1
Approx. cubic	0.991	5.0	4.0	3.3	2.7

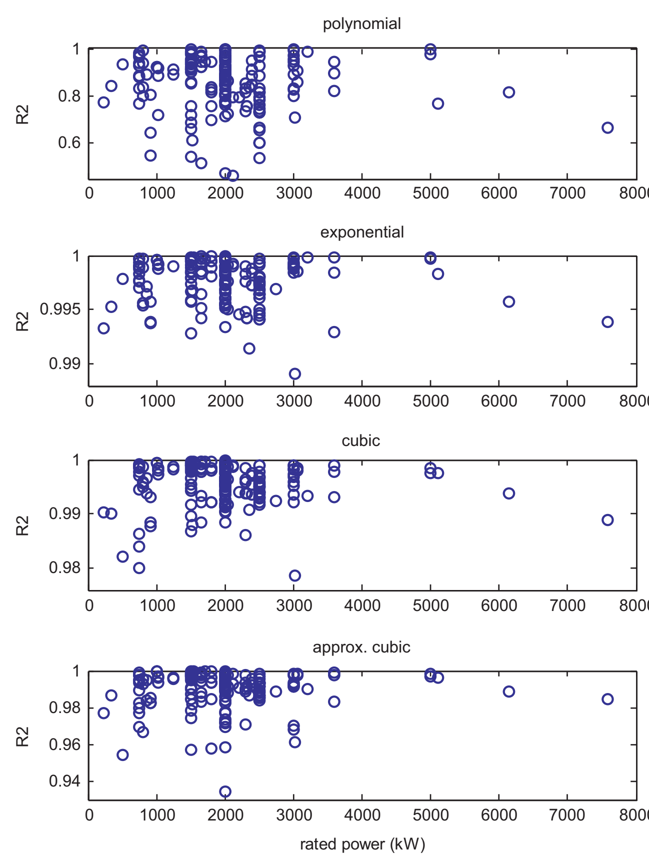


Fig. 9. Values obtained for the coefficient of determination R^2.

Table 4
Summary of R^2 values: mean and standard deviation.

Approximation	Mean R^2	Std R^2
Polynomial	0.8337	0.1673
Exponential	0.9978	0.0020
Cubic	0.9956	0.0038
Approx. cubic	0.9903	0.0100

curve (polynomial, exponential, cubic or approximate cubic power curve).

6. Results

All equations for power curve modelling have been applied for each power curve in the database. As an example, a power curve of a 2 MW VSWT, whose manufacturer power curve is shown in Table 1, has been analysed. The approximations curves are presented in Fig. 8 and its parameters can be seen in Table 2. For this case, the obtained results are shown in Table 3.

Fig. 9 represents the R^2 results from fitting the power curves of all the VSWTs from the database. As can be seen, the R^2 values for the polynomial fitting are the worst with values lower than 0.5. This is because the polynomial expression is the one that depends the most on data presented by the manufacturer in the power curve. Also, it can be seen in Fig. 9 that R^2 values for the exponential, cubic and the approximate cubic are over the 0.92 which is a pretty good fit. So far, it can be concluded that the exponential and the cubic have the best behaviour. Table 4 shows a summary of the mean and standard deviation values of the results obtained for R^2.

The distributions of errors of energy density ε for the polynomial, exponential, cubic and the approximate cubic power curves are shown in Fig. 10. The main conclusion here is that the exponential and the cubic power curves, represented by (7) and (8), have the best behaviour in terms of mean power error and the lowest standard deviation. Results can be seen in Table 5.

Polynomial approximation has the worst results for all goodness of fit indicators. This can be explained by its strong dependence on the power curve parameters: cut-in, cut-out and rated wind speed, specially the last one. The effect that the rated wind speed value has in the fitting results is analysed in Appendix A.

7. Conclusions

A review of the most common equations (polynomial, exponential, cubic and approximate cubic) used to model VSWT power curves has been presented. They have been analysed in order to establish their capability to represent commercial VSWTs. For this purpose, data from nearly 200 commercial VSWTs has been used. The comparison between power curve models has been done using the well-known coefficient of determination R^2. Furthermore, for comparison purposes, it has been introduced the difference between the estimation of the energy production using manufactured power curve and using its approximation. For the sake of simplicity, a set reduced set of Rayleigh wind distributions has been considered for these energy estimations.

Finally, the results of evaluating the power curve modelling methods can be summarised as follows:

- Exponential and cubic equations are the best when the coefficient of determination and the error in energy density are considered.

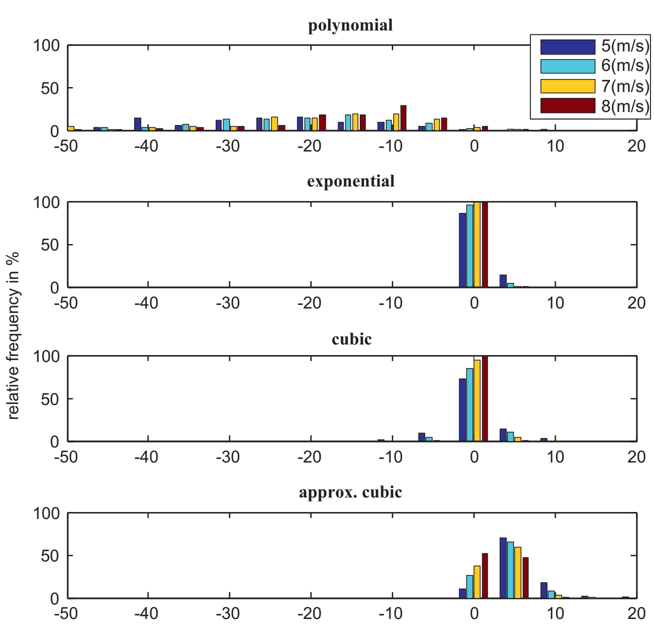


Fig. 10. Distribution of mean power error for the polynomial, exponential, cubic and cubic approx. expressions at different mean wind speeds (5, 6, 7, and 8 m/s).

Table 5
Summary of ε values: mean and standard deviation.

Approximation		Mean wind speed of Rayleigh distance			
		5 m/s	6 m/s	7 m/s	8 m/s
Polynomial	Mean ε	−0.2862	−0.2302	−0.1873	−0.1553
	Std ε	0.1623	0.1415	0.1225	0.1063
Exponential	Mean ε	0.0124	0.0087	0.0069	0.0058
	Std ε	0.0129	0.0082	0.0064	0.0055
Cubic	Mean ε	0.0065	0.0047	0.0042	0.0039
	Std ε	0.0280	0.0168	0.0111	0.0080
Approx. cubic	Mean ε	0.0542	0.0419	0.0336	0.0276
	Std ε	0.0298	0.0224	0.0183	0.0153

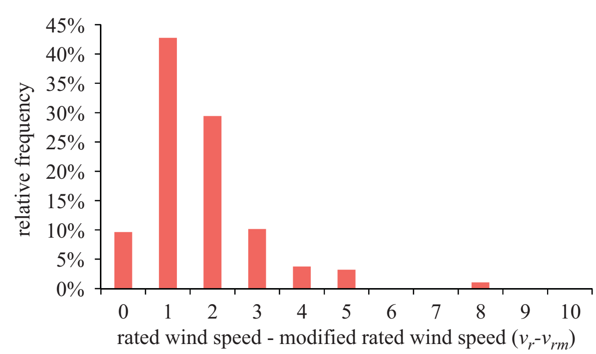


Fig. 11. Histogram of differences between rated wind speed and modified rated wind speed ($v_r - v_{rm}$).

Table 6
Summary of R^2 values with v_{rm}.

Approximation	Mean R^2	STD R^2
Polynomial	0.9820	0.0117

Table 7
Summary of ε values with v_{rm}.

Approximation		Mean wind speed of Rayleigh distance			
		5 m/s	6 m/s	7 m/s	8 m/s
Polynomial	Mean ε	−0.0224	−0.0100	−0.0043	−0.0015
	Std ε	0.1623	0.1415	0.1225	0.1063

Table 8
Wind turbine database.

Manufacturer	Model	Power (kW)	Diameter (m)	Cut-in speed (m/s)	Cut-out speed (m/s)	Rated speed (m/s)	Maximum C_p
AAER	A1650-77 LM	1650	77.0	3.5	20.0	12.0	0.44
AAER	A1650-77 AB	1650	77.0	3.5	20.0	12.0	0.45
AAER	A1650-82	1650	82.0	3.5	20.0	12.0	0.46
ACSA	A27/225	225	27.0	3.5	25.0	13.5	0.46
Alizeo	Alizeo 1000/56	1000	56.0	3.5	25.0	11.0	0.37
Alizeo	Alizeo 1000/60	1000	60.0	3.5	25.0	11.0	0.38
Alizeo	Alizeo 1000/60 DD	1000	60.0	3.5	15.0	11.0	0.45
Alizeo	Alizeo 1000/64	1000	64.0	3.5	25.0	11.0	0.38
Alizeo	Alizeo 1000/64 DD	1000	60.0	3.5	15.0	11.0	0.45
Areva	M5000	5000	116.0	4.0	25.0	12.5	0.42
AVIC Huide	HD2000	2000	93.2	3.5	25.0	12.0	0.43
Clipper	Liberty C89	2500	89.0	4.0	25.0	13.3	0.42
Clipper	Liberty C93	2500	93.0	4.0	25.0	12.9	0.42
Clipper	Liberty C96	2500	96.0	4.0	25.0	12.8	0.43
Clipper	Liberty C99	2500	99.0	4.0	25.0	12.6	0.42
Clipper	Liberty C100	2500	100.0	3.0	25.0	14.0	0.42
Dewind	D8.2	2000	80.0	4.5	25.0	15.0	0.38
Dewind	D9	2000	93.0	3.0	25.0	12.0	0.44
Dewind	D9.1	2000	93.0	3.0	25.0	12.0	0.48
Dewind	D9.2	2000	93.0	4.5	25.0	12.0	0.41
Doosan	WinDS3000	3000	91.3	3.0	25.0	13.0	0.48
e.n.o. energy	EE82	2000	82.4	3.0	25.0	13.0	0.44
e.n.o. energy	EE92	2200	92.8	3.0	25.0	13.0	0.47
Enercon	E33/330	330	33.4	3.0	28.0	12.6	0.50
Enercon	E44/900	900	44.0	3.0	28.0	15.5	0.50
Enercon	E48/800	800	48.0	3.0	28.0	13.2	0.50
Enercon	E53/800	800	52.9	2.0	28.0	12.5	0.49
Enercon	E70/2300	2300	71.0	2.0	28.0	14.5	0.50
Enercon	E82/2000	2000	82.0	2.0	28.0	12.4	0.50
Enercon	E70/E4	2300	71.0	2.5	28.0	14.0	0.50
Enercon	E126/7500	7500	127.0	3.0	28.0	15.5	0.48
Enercon	E82/2300	2300	82.0	2.0	28.0	13.5	0.50
Enercon	E101/3000	3000	101.0	2.0	28.0	11.7	0.48
Enercon	E82/3000	3000	82.0	2.0	28.0	16.1	0.50
Enron	750/50	750	50.0	3.0	29.0	11.2	0.45
Enron	750/48	750	48.0	3.5	29.0	11.6	0.46
Enron	750/46	750	46.0	4.5	29.0	12.1	0.46
Eviag	ev100	2500	100.0	3.5	25.0	11.5	0.46
Eviag	ev2.93	2050	93.2	3.5	25.0	12.0	0.46
Eviag	ev90	2500	90.0	4.0	25.0	13.0	0.45
EWT	Directwind 54/900	900	54.0	2.5	25.0	13.0	0.48
EWT	Directwind 52/750	750	51.5	2.5	25.0	13.0	0.45
EWT	Directwind 52/900	900	51.5	2.5	25.0	13.0	0.47
Fuhrlander	FL MD/70	1500	70.0	3.0	25.0	11.6	0.45
Fuhrlander	FL MD/77	1500	77.0	3.0	20.0	13.0	0.47
Fuhrlander	FL 1500/70	1500	70.0	3.0	25.0	12.0	0.49
Fuhrlander	FL 1500/77	1500	77.0	3.0	20.0	11.0	0.47
Gamesa	G52/850	850	52.0	4.0	28.0	15.0	0.46
Gamesa	G58/850	850	58.0	3.0	23.0	12.0	0.45
Gamesa	G80/2000	2000	80.0	4.0	25.0	15.0	0.43
Gamesa	G87/2000	2000	87.0	4.0	25.0	15.0	0.45
Gamesa	G90/2000	2000	90.0	3.0	25.0	14.0	0.45
GE Energy	1.5sl	1500	77.0	3.5	20.0	12.0	0.43
GE Energy	2.5xl 100	2500	100.0	3.0	25.0	12.0	0.39
GE Energy	2.5xl 103	2500	103.0	3.0	25.0	12.0	0.43
Ghodawat	G1650/77	1650	77.4	3.5	20.0	11.0	0.41
Ghodawat	G1650/82	1650	82.0	3.5	20.0	12.0	0.44
Global Wind Power	GWP82-2000	2000	82.5	2.7	25.0	12.5	0.45
Goldwind	GW77/1500	1500	77.0	3.0	22.0	11.0	0.45
Goldwind	GW70/1500	1500	70.0	3.0	25.0	11.8	0.45
Goldwind	GW82/1500	1500	82.0	3.0	22.0	10.3	0.47
Guangdong Mingyang	MY1.5s	1500	77.4	3.0	25.0	11.0	0.41
Guangdong Mingyang	MY1.5se	1500	82.7	3.0	25.0	10.5	0.41
Guodian	UP77	1500	77.4	3.0	25.0	11.1	0.44
Guodian	UP82	1500	82.8	3.0	25.0	10.5	0.45
Guodian	UP86	1500	86.1	3.0	25.0	10.0	0.46
Hyosung	HS90-2 MW	2000	90.6	4.0	25.0	11.0	0.41
Hyosung	HS50-750 MW	750	50.0	3.5	25.0	12.0	0.42
Hyundai	AV928	2500	93.0	3.0	25.0	11.5	0.50
HZ Windpower	H82-2000	2000	82.4	4.0	25.0	12.4	0.43
HZ Windpower	H87-2000	2000	87.0	3.5	25.0	11.8	0.33
HZ Windpower	H93-2000	2000	92.8	3.0	25.0	11.0	0.43
IMPSA	IWP-70-1500	1500	70.0	4.0	25.0	13.0	0.42
IMPSA	IWP-83-2100	2100	83.0	3.0	25.0	13.0	0.45
IMPSA	IWP-93-2100	2100	93.0	3.0	25.0	13.0	0.45

Table 8 (*continued*)

Manufacturer	Model	Power (kW)	Diameter (m)	Cut-in speed (m/s)	Cut-out speed (m/s)	Rated speed (m/s)	Maximum C_p
Inox Wind	WT 2000 DF	2000	93.0	3.0	20.0	11.5	0.44
JSW	J82	2000	83.3	3.5	25.0	13.0	0.46
Lagerwey	L82	2000	82.5	2.7	28.0	12.5	0.46
Leitwind	LTW70-1700	1700	70.1	3.0	25.0	12.0	0.43
Leitwind	LTW70-2000	2000	70.1	3.0	25.0	13.0	0.43
Leitwind	LTW77-1500	1500	76.6	3.0	25.0	12.0	0.48
Leitwind	LTW80-1500	1500	80.3	3.0	25.0	10.5	0.50
Leitwind	LTW80-1800	1800	80.3	3.0	25.0	11.3	0.44
M Torres	MT TWT 82/1650	1650	82.0	3.0	25.0	13.0	0.44
M Torres	MT TWT 70/1650	1650	70.0	3.0	25.0	13.9	0.43
M Torres	MT TWT 77/1650	1650	77.0	3.0	25.0	13.5	0.43
Made	AE-52	800	52.0	3.5	25.0	11.9	0.41
Made	AE-56	800	56.0	3.3	25.0	11.6	0.48
Mitsubishi	MWT-92-2.3	2300	92.0	3.0	25.0	12.5	0.43
Mitsubishi	MWT-100-2.4	2400	100.0	3.0	25.0	12.0	0.42
Mitsubishi	MWT-102-2.4	2400	102.0	3.0	25.0	11.5	0.42
Mitsubishi	MWT-95	2400	95.0	3.0	25.0	12.5	0.43
Mitsubishi	MWT-92	2400	92.0	3.0	25.0	12.5	0.43
Neg Micon	NM92/2750	2750	92.0	4.0	25.0	14.0	0.45
Nordex	N90/2500 HS	2500	90.0	3.0	25.0	13.0	0.46
Nordex	N90/2500 LS	2500	90.0	3.0	25.0	14.0	0.46
Nordex	N90/2300	2300	90.0	3.0	25.0	13.0	0.44
Nordex	N90/2500 HS offshore	2500	90.0	3.0	25.0	13.0	0.46
Nordex	N90/2500 LS offshore	2500	90.0	3.0	25.0	14.0	0.46
Nordex	S82	1500	82.0	3.5	25.0	12.5	0.44
Nordex	N100/2500	2500	100.0	3.0	20.0	12.5	0.45
Nordex	N80/2500	2500	80.0	3.0	25.0	15.0	0.43
PowerWind	PW90	2500	90.0	3.0	25.0	14.0	0.46
PowerWind	PW100	2500	100.0	3.0	25.0	13.0	0.45
Repower	5M on shore	5075	126.0	3.5	25.0	14.0	0.45
Repower	5M off shore	5075	126.0	3.5	30.0	14.0	0.45
Repower	MM92	2050	92.5	3.0	24.0	12.5	0.46
Repower	MM82	2050	82.0	3.5	25.0	14.5	0.44
Repower	6M on shore	6150	126.0	3.5	25.0	15.0	0.42
Repower	6M off shore	6150	126.0	3.5	30.0	14.0	0.42
Repower	3.2M114	3200	114.0	3.0	22.0	12.0	0.34
Repower	MM100	1800	100.0	3.0	22.0	11.0	0.45
Sany	SE8220III	2000	82.5	3.5	25.0	12.8	0.36
Sany	SE8720III	2000	87.0	3.5	25.0	11.7	0.51
Sany	SE8720III-60 Hz	2000	87.0	3.5	25.0	11.7	0.44
Sany	SE9320III-3	2000	93.0	3.0	22.0	10.8	0.57
Sany	SE9320III-S3	2000	93.0	3.5	22.0	12.5	0.44
Sany	SE11030III-S	3000	110.0	3.5	25.0	13.0	0.43
Shandong Swiss Electric	YZ78/1.5	1500	78.0	2.3	25.0	10.5	0.48
Shandong Swiss Electric	YZ82/1.5	1500	82.0	2.3	20.0	10.2	0.43
Shandong Swiss Electric	YZ87/2.0	2000	87.0	2.3	20.0	11.0	0.52
Shandong Swiss Electric	YZ88/2.5	2500	88.0	3.0	25.0	12.0	0.50
Shandong Swiss Electric	YZ90/2.5	2500	90.0	2.8	22.0	11.0	0.48
Shandong Swiss Electric	YZ113/3.0	3000	113.0	2.5	20.0	10.5	0.45
Shanghai Electric	W1250/62	1250	62.3	2.5	25.0	12.5	0.44
Shanghai Electric	W1250/64	1250	64.3	3.0	25.0	12.3	0.45
Shanghai Electric	W2000/87	2000	87.0	3.0	25.0	12.4	0.42
Shanghai Electric	W2000/93	2000	93.0	3.0	25.0	12.0	0.42
Shanghai Electric	W2000/99	2000	99.0	3.0	25.0	11.8	0.42
Shanghai Electric	W2000/105	2000	105.0	3.0	20.0	11.5	0.43
Shanghai Electric	W3600/116	3600	116.0	3.5	25.0	12.0	0.44
Shanghai Electric	W3600/122	3600	122.0	3.5	25.0	11.5	0.42
Siemens	SWT-3.6-107	3600	107.0	4.0	25.0	13.0	0.44
Siemens	SWT-2.3-82 VS	2300	82.4	4.0	25.0	13.0	0.42
Siemens	SWT-2.3-93	2300	93.0	4.0	25.0	13.0	0.47
Siemens	SWT-2.3-101	2300	101.0	4.0	25.0	12.0	0.42
Sinovel	SL 3000/90	3000	91.6	3.5	25.0	13.0	0.46
Sinovel	SL 3000/100	3000	101.2	3.5	25.0	12.5	0.46
Sinovel	SL 1500/70	1500	70.4	3.0	25.0	12.0	0.42
Sinovel	SL 1500/77	1500	77.4	3.0	20.0	11.0	0.42
Sinovel	SL 1500/82	1500	82.9	3.0	20.0	10.5	0.44
STX Windpower	STX72	2000	70.7	3.0	20.0	13.0	0.43
STX Windpower	STX82 1.5 MW	1500	82.7	3.0	25.0	12.0	0.39
STX Windpower	STX82 2.0	2000	82.7	3.0	25.0	12.5	0.42
STX Windpower	STX93 2.0	2000	93.3	3.0	20.0	11.0	0.43
Unison	U93	2000	93.0	3.0	25.0	11.5	0.42
Unison	U88	2000	88.0	3.0	25.0	12.0	0.42
Unison	U50	750	50.0	3.0	25.0	12.5	0.40
Unison	U54	750	54.0	3.0	25.0	11.5	0.43
Unison	U57	750	57.0	3.0	25.0	11.5	0.48
Vensys	V77	1500	77.0	3.0	22.0	13.0	0.46

Table 8 (*continued*)

Manufacturer	Model	Power (kW)	Diameter (m)	Cut-in speed (m/s)	Cut-out speed (m/s)	Rated speed (m/s)	Maximum C_p
Vensys	V70	1500	70.0	3.0	25.0	13.5	0.43
Vensys	V100	2500	100.0	3.0	25.0	11.0	0.45
Vensys	V82	1500	82.0	3.0	22.0	12.5	0.45
Vensys	V90	2500	90.0	3.0	25.0	12.0	0.44
Vestas	V80/2000	2000	80.0	4.0	25.0	15.0	0.44
Vestas	V80/2000 offshore	2000	80.0	4.0	25.0	16.0	0.45
Vestas	V80/2000 grids	2000	80.0	4.0	25.0	14.5	0.44
Vestas	V90/1800	1800	90.0	4.0	25.0	12.0	0.38
Vestas	V90/1800 grids	1800	90.0	4.0	25.0	13.0	0.44
Vestas	V90/2000	2000	90.0	4.0	25.0	12.0	0.42
Vestas	V90/2000 grids	2000	90.0	4.0	25.0	13.5	0.46
Vestas	V112/3000 offshore	3000	112.0	3.0	25.0	12.5	0.46
Vestas	V112/3000	3000	112.0	3.0	25.0	13.0	0.43
Vestas	V100/1800	1800	100.0	3.0	20.0	12.0	0.42
Vestas	V100/1800 grids	1800	100.0	3.0	20.0	12.0	0.48
Wikov	W2000/86	2000	86.4	3.5	25.0	12.0	0.50
Wikov	W2000/93	2000	93.0	3.0	20.0	11.0	0.50
Windflow	Windflow 500	500	33.2	5.5	30.0	13.7	0.50
Windtec	WT1650df/77	1650	77.0	3.5	20.0	12.0	0.46
Windtec	WT1650df/82	1650	82.0	3.5	20.0	12.0	0.47
Windtec	WT2000df/86	2000	86.0	3.5	25.0	11.3	0.45
Windtec	WT2000fc/86	2000	86.0	3.5	25.0	11.3	0.45
Windtec	WT2000fc/93	2000	93.0	3.0	20.0	11.0	0.45
Windtec	WT2000df/93	2000	93.0	3.0	20.0	11.0	0.45
Windtec	WT3000fc/91	3000	91.3	3.5	25.0	13.0	0.46
Windtec	WT3000sg/91	3000	91.3	4.0	25.0	13.0	0.46
Windtec	WT3000fc/100	3000	100.0	3.5	25.0	12.5	0.48
Windtec	WT3000sg/100	3000	100.0	3.5	25.0	12.5	0.48
Winwind	WinWind 3-109	3000	109.0	4.0	25.0	11.0	0.47
Winwind	WinWind 3-120	3000	120.0	4.0	25.0	11.0	0.45
Xemc-Darwind	5 MW DD115 offshore	5000	115.0	4.0	25.0	12.0	0.39
WindEnergy Lebanon	750H_1	750	50.0	3.0	25.0	12.5	0.47
WindEnergy Lebanon	750H_2	750	54.0	3.0	25.0	11.5	0.46
WindEnergy Lebanon	750H_3	750	57.0	3.0	25.0	11.5	0.49
WindEnergy Lebanon	2 MW/88	2000	88.0	3.0	25.0	12.0	0.41
Samsung	25s	2500	90.0	3.5	25.0	12.8	0.46

- Approximate cubic equation gives acceptable values of goodness of fit which makes this approximation very attractive due to its simplicity.
- Polynomial equation, in spite of how simply are obtained its parameters, gives the worst results in terms of fitting. This is caused by its sensitivity to the values of the parameters, especially to the rated wind speed value.

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Appendix A. The effect of rated wind speed value in polynomial approximation

The polynomial approximation is calculated by using cut-in, cut-out and rated wind speeds. Its behaviour, in terms of fitting, strongly depends on the values of these parameters, especially on rated wind speed. In order to analyse this behaviour a new value for the rated wind speed called modified rated wind speed (v_{rm}) has been computed. v_{rm} has been calculated so that its value minimises the index J (see (10)). The histogram of differences between the rated wind speed and the modified rated wind speed is shown in Fig. 11.

Results in Tables 6 and 7 show that the values of indicators of goodness of fit for polynomial approximation have improved when v_{rm} has been considered. However this improvement in the indicators is not enough to achieve the values shown for the rest of approximations (see Tables 4 and 5). Moreover, the main advantage of polynomial approximation is how simply its parameters are obtained. Nevertheless, that simplicity disappears when the parameter v_{rm} needs to be calculated.

For the other approximation equations, the improvement achieved by using a similar procedure is negligible.

Appendix B. Wind turbine database

Table 8 shows the main values of the database of wind turbines.

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